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ON THE PROBLEM OF THE ELASTIC STABILITY OF A LOCALLY LOADED CYLINDRICAL SHELL (SUPPLEMENT)

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An approximate analytic representation of the solution of a nonlinear equation describing the subcritical axisymmetric shell bending was used in [1] in an investigation of the stability of equilibrium of a semi-infinite circular cylindrical shell loaded by a uniform radial stress resultant along a hinge supported edge. In substance, this representation corresponds to the two first terms of the expansion of the desired nonlinear solution in a power series of the parameter

$$p = \mu q = 2Q / (Eh\mu^2) \, (\mu^2 = h / [R \sqrt{3(1-\nu^2)}])$$

Here Q is the intensity of the external radial stress resultant, h and R are the shell thickness and radius, and E and ν are the Young's modulus and Poisson's ratio of the shell material. The construction of higher approximations was nor carried out because of their extreme awkwardness.

However, the desire to solve more exactly the stability problem formulated in [1] forced the authors to return to the question of refining the solution of the nonlinear boundary value problem of subcritical shell bending. To solve this problem, the procedure of differentiating with respect to the parameter was used in combination with the method of finite differences.

Differentiating (1.1) from [1] with respect to the parameter p and later going over to finite differences yields the following successive approximations process to the desired nonlinear solution (the meaning of the notation is disclosed in [1]): if the functions $\eta_i(x)$ and $\vartheta_i(x)$ are a solution of the nonlinear problem for $p = p_i$, then the functions

$$\eta_{i+1}(x) = \eta_i(x) + \Delta p \eta_i^{\cdot}(x), \quad \vartheta_{i+1}(x) = \vartheta_i(x) + \Delta p \vartheta_i^{\cdot}(x)$$

are the approximate solution for $p = p_{i+1} = p_i + \Delta p$, where $\eta_i^{\cdot}(x)$ and $\vartheta_i^{\cdot}(x)$ are determined as a result of solving the linear boundary value problem

$$\eta_i^{\cdot\prime\prime} + 2\vartheta_i^{\cdot} \cos\vartheta_i = 0, \qquad \vartheta_i^{\cdot\prime\prime} + 2\vartheta_i^{\cdot}\eta_i \sin\vartheta_i - 2\eta_i^{\cdot} \cos\vartheta_i = 0 \quad (0 \leqslant x \leqslant \infty)$$

$$\eta_i^{\cdot} = 1, \qquad \vartheta_i^{\cdot\prime} = 0 \quad (x = 0), \qquad \eta_i^{\cdot} = \vartheta_i^{\cdot} = 0 \quad (x = \infty)$$

Since there is an exact solution $\eta_0 \equiv \vartheta_0 \equiv 0$ for $p = p_0 = 0$, then starting therefrom and selecting a sufficiently small step Δp for the change in the parameter p, we can obtain an approximate solution of the nonlinear problem for a number of values of this parameter.

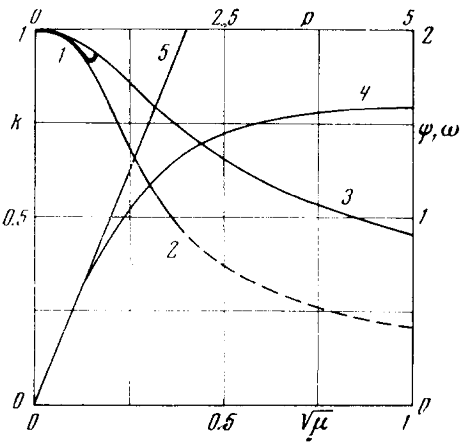

Fig. 1

The process described was realized on an electronic digital computer where the linear boundary value problem governing the increment in the solution, when going from the preceding to the succeeding step, was solved by the factorization method in finite differences. The stability problem for the axisymmetric equilibrium state corresponding to the nonlinear solution obtained was formulated and solved as in [1].

The calculations were performed in the $0 \leqslant p \leqslant 7$ range. The magnitudes of the steps in the parameter p and in the coordinate were varied in order to verify the accuracy. The final results are presented in Fig. 1.

Curves 3 and 4 determine values of the parameters ψ and ω as a function of p, which are the characteristics of the shell subcritical state. In particular, the greatest value of the ring stress is calculated by using ψ ; $\sup |\sigma_2| \simeq E\,\mu p\psi / 2$, and the greatest value of the angle of rotation by using ω : $\sup |\vartheta| \simeq \omega$. Both these values are achieved on the shell edge. It is curious that the values of ω determined by the curve 4 for $p > 3$ are close to $\pi / 2$, while the solution of the linear axisymmetric bending problem results in values of ω in the form of the line 5. The domain of elastic strain of the shell can be determined by means of the greatest value of the ring stress. Curve 2 pictures the dependence of the critical value of the load parameter $k = Q^* / Q^\circ$ ($Q^\circ \simeq 16.35\, Eh\mu^3$ is the critical value of the external stress resultant in determining the subcritical state by linear theory) on $\sqrt{\mu}$. Presented as curve 1 for comparison (heavy line) is the corresponding dependence from [1]. The dashed section of the curve 2 emerges beyond the limits of the domain of definition of thin shells by the inequality $h / R \leqslant 1 / 20$. It should be treated as a formal continuation of the solution of the considered eigenvalue problem in the parameter μ. The substantial discrepancy between curve 1 corresponding to the solution of the nonlinear problem in a first approximation [1] and curve 2 corresponding to the refined solution of the nonlinear problem, starts with the value $\sqrt{\mu} \simeq 0.13$. However, the main deduction of [1] remains valid: linearizing the subcritical state is admissible only for very small values of the ratio h / R. In particular, for $h / R \leqslant 1.74 \cdot 10^{-4} \sqrt{3(1 - \nu^2)}$ the error in the magnitude of the critical load because of linearization does not exceed 5%. Outside this domain the error grows rapidly as the ratio h / R increases.

The graphical dependences presented in [2] can also be refined by using the results presented here.

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SOLUTION OF THE AXISYMMETRIC PROBLEM OF THE THEORY OF ELASTICITY FOR TRANSVERSELY ISOTROPIC BODIES WITH THE AID OF GENERALIZED ANALYTIC FUNCTIONS

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The solution of the axisymmetric problem for transversely isotropic bodies, expressed in terms of generalized analytic functions, is constructed. We obtain formulas for the displacements and stresses, similar to the corresponding formulas of the plane problem. The representation of the generalized analytic functions by analytic ones, are indicated, and the analogue of the Cauchy-type integral which gives the possibility of reducing the boundary value problems to integral equations, is presented. As an example, we consider the action of forces which are distributed along a circumference in the interior of a transversely isotropic space.

The plane problems of the theory of elasticity for transversely isotropic bodies are solved effectively with the aid of analytic functions of a complex variable [1]. In [2, 3] the solution of axisymmetric and nonaxisymmetric problems for bodies of revolution with the aid of analytic functions and contour integrals, was considered. In the case of an isotropic elastic medium, the solution of axisymmetric problems with the aid of a class of generalized analytic functions [4] was proposed.

1. Let $U_k(z, r)$ and $V_k(z, r)$ be complex functions satisfying the system of equations

$$\gamma_k \frac{\partial U_k}{\partial z} = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) V_k, \qquad \frac{\partial U_k}{\partial r} = -\gamma_k \frac{\partial V_k}{\partial z} \qquad (k = 1; 2) \tag{1.1}$$

where the parameter γ_k is some number, in general, complex. These functions, obviously, satisfy the differential equations

$$\Delta_k U_k = 0, \qquad \left(\Delta_k - \frac{1}{r^2}\right) V_k = 0$$
$$\left(\Delta_k = \gamma_k^2 \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) \tag{1.2}$$